Q1a

1a

(a) Evaluate

$$4\log_3 729 + 3\log_2 64^2 - 3\log 100 + \ln e^6$$

3^6 $(2^6)^2$ 10^2 6

2^{12}

[2]

(b) Evaluate

$$\tfrac{1}{2}\ln 196 + \tfrac{1}{3}\ln 125 + \tfrac{1}{4}\ln 81 + \tfrac{1}{5}\ln 32$$

giving your answer in the form $\ln q$.

[3]

a) Rewrite the numbers inside the functions as powers of their bases:

$$4\log_3 3^6 + 3\log_2 2^{12} - 3\log 10^2 + 6$$

Simplify,

$$= 4(6) + 3(12) - 3(2) + 6$$

$$= \boxed{60}$$

Q1b

1b

(a) Evaluate

$$4\log_3 729 + 3\log_2 64^2 - 3\log 100 + \ln e^6$$

[2]

(b) Evaluate

$$\tfrac{1}{2}\ln 196 + \tfrac{1}{3}\ln 125 + \tfrac{1}{4}\ln 81 + \tfrac{1}{5}\ln 32$$

giving your answer in the form $\ln q$.

[3]

b) Rewrite the coefficients as powers

$$\ln\sqrt{196} + \ln\sqrt[3]{125} + \ln\sqrt[4]{81} + \ln\sqrt[5]{32}$$

$$= \ln 14 + \ln 5 + \ln 3 + \ln 2$$

Combine the terms using log law of addition

$$= \ln(14 \times 5 \times 3 \times 2)$$

$$= \boxed{\ln 420}$$

Q2

2

Solve the equation

$$2 \times 5^{2x+1} + 21 = 41 \times 5^x,$$

giving your answers in the form $\log_a b$, where a and b are rational numbers to be found.

$5^{2x+1} = 5(5^x)^2$

[4]

Spot the hidden quadratic!

$2(5)(5^x)^2 - 41(5^x) + 21 = 0$

Let $y = 5^x$

$10y^2 - 41y + 21 = 0$

$(5y-3)(2y-7) = 0$

$y = \frac{7}{2}, \frac{3}{5}$

$5^x = \frac{7}{2}, \quad 5^x = \frac{3}{5}$

$\boxed{x = \log_5 \frac{7}{2} \quad x = \log_5 \frac{3}{5}}$

Q3a

3a

Solve the following equations, giving your answers correct to 3 significant figures.

(a) $8e^{3x^2-1} = 12$

[3]

(b) $e^{3x} - 42 = 2e^x(6e^x - 7)$

[3]

a) $e^{3x^2-1} = \frac{12}{8} = \frac{3}{2}$

$\ln e^{3x^2-1} = \ln\frac{3}{2}$

$3x^2 - 1 = \ln\frac{3}{2}$

$x = \sqrt{\frac{\ln\frac{3}{2}+1}{3}}$

$= \boxed{0.684, -0.684}$ (3sf)

Q3b

3b

Solve the following equations, giving your answers correct to 3 significant figures.

(a) $8e^{3x^2-1} = 12$

[3]

(b) $e^{3x} - 42 = 2e^x(6e^x - 7)$

[3]

b) $e^{3x} - 42 = 12(e^x)^2 - 14e^x$

Spot the hidden cubic!

$(e^x)^3 - 42 = 12(e^x)^2 - 14(e^x)$

$(e^x)^3 - 12(e^x)^2 + 14e^x - 42 = 0$

Let $y = e^x$

$y^3 - 12y^2 + 14y - 42 = 0$

$y = 11.0$

$e^x = 11.0$

$x = \ln 11.0 = \boxed{2.41}$ (3sf)

Q4

4

Show that

$$2\log_3 x + \log_3(x^2 - 1) - 2\log_3(x + 1) \equiv \log_3 \frac{x^2(x-1)}{(x+1)}.$$

[3]

Rewrite coefficients as powers

LHS: $\log_3 x^2 + \log_3(x^2-1) - \log_3(x+1)^2$

difference of two squares

$= \log_3 \frac{x^2(x^2-1)}{(x+1)^2} = \log_3 \frac{x^2\cancel{(x+1)}(x-1)}{(x+1)^{\cancel{2}\,1}}$

$= \log_3 \frac{x^2(x-1)}{x+1} = RHS$

$\boxed{\therefore LHS = RHS}$

Q5

5

Write the following as a single logarithm

$$2\log_p(x+1) + 3\log_p(x-1) - \log_p(x^2-1).$$

[3]

Rewrite coefficients as powers

$\log_p(x+1)^2 + \log_p(x-1)^3 - \log_p(x^2-1)$

Combine using log laws of addition & subtraction

$\log_p \dfrac{(x+1)^2(x-1)^3}{x^2-1}$ difference of 2 squares!

$\log_p \dfrac{(x+1)^{\cancel{2}\,1}(x-1)^{\cancel{3}\,2}}{\cancel{(x+1)}\cancel{(x-1)}} = \boxed{\log_p(x+1)(x-1)^2}$

Q6

6

On the same axes, sketch the graphs of $y = e^{2x}$ and $y = \frac{1}{2}\ln x$.
On each graph, label any points where the graph intersects the coordinate axes.
Write down the equations of any asymptotes for each graph.
Explain the significance of the line $y = x$.

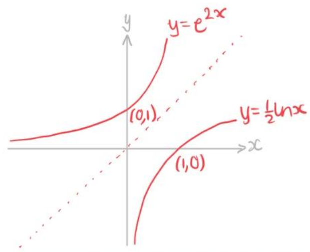
[5]

compression in x direction

$\rightarrow 2 \leftarrow$

stretch in y direction

$\uparrow \frac{1}{2} \downarrow$



$y = x$ is the line of reflection

(the functions are the inverse of one another, this can be shown mathematically too!)

$y = e^{2x}$

$\ln y = 2x$

$x = \frac{1}{2}\ln y$

reflect along $y = x$

$y = \frac{1}{2}\ln x$

asymptote for $y = e^{2x}$ is $y = 0$
asymptote for $y = \frac{1}{2}\ln x$ is $x = 0$

Q7

7

Show that $4 - \ln 16$ can be written in the form $4 \ln\left(\frac{e}{2}\right)$.

2^4

$4(\ln e - \ln 2)$

1

[3]

$4 - \ln 2^4$

$= 4 - 4\ln 2$

$= 4(1 - \ln 2)$

by writing 1 as $\ln e$

$= 4(\ln e - \ln 2)$

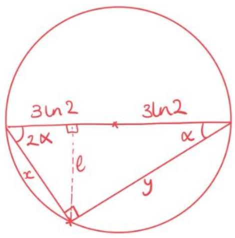
$= \boxed{4 \ln\left(\frac{e}{2}\right)}$

Q8

8

A triangle is drawn inside a circle such that one side of the triangle is the diameter and all three vertices of the triangle lie on the circumference.
The radius of the circle is $(3 \ln 2)$ cm.
The two smallest angles in the triangle are α and β respectively where $\beta = 2\alpha$.
Find all three sides of the triangle, giving your answers in the form $a \ln 2$.

[5]



circle theorem:
angles subtended by diameter are 90°.

$180° = 90° + 3\alpha$

$\alpha = 30°$

pythagoras' theorem

$(6\ln 2)^2 = x^2 + y^2$ ①

trig!

$\sin 2\alpha = \frac{l}{x}$ $\sin \alpha = \frac{l}{y}$

eliminate l

$x \sin 2\alpha = y \sin \alpha$

② $y = \frac{x \sin 2(30)}{\sin 30} = \sqrt{3}\, x$

solve simultaneous eqns ①&②

sub $y = \sqrt{3}x$ into ①

$(6\ln 2)^2 = x^2 + (\sqrt{3}x)^2 = 4x^2$

$x = \frac{6\ln 2}{2} = 3\ln 2$

sub $x = 3\ln 2$ into ②

$y = \sqrt{3}(3\ln 2) = 3\sqrt{3}\ln 2$

3 sides: $\boxed{6\ln 2,\ 3\ln 2,\ 3\sqrt{3}\ln 2}$

Q9

9

How many real solutions does the equation have? Justify your answer.

$$3\log_x(x+1) = \ln e^3$$

3

[3]

$\log_x(x+1)^3 = 3$

$x^3 = (x+1)^3$

$= (x+1)(x^2+2x+1)$

$\cancel{x^3} = \cancel{x^3} + 3x^2 + 3x + 1$

$0 = 3x^2 + 3x + 1$

discriminant

$b^2 - 4ac = 3^2 - 4(3)(1) = -3$

$-3 < 0$

NO REAL SOLUTIONS
since discriminant < 0

Q10

10

Without using a calculator, show that

$$\log_4 8 = \log_9 27.$$

[3]

LHS

$8 = 2^3 = (\sqrt{4})^3 = (4^{\frac{1}{2}})^3 = 4^{\frac{3}{2}}$

$\log_4 4^{\frac{3}{2}}$

$= \frac{3}{2}\log_4 4$ 1

$= \frac{3}{2}$

RHS

$27 = 3^3 = (\sqrt{9})^3 = (9^{\frac{1}{2}})^3 = 9^{\frac{3}{2}}$

$\log_9 9^{\frac{3}{2}}$

$= \frac{3}{2}\log_9 9$ 1

$= \frac{3}{2}$

$\therefore$ LHS = RHS